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Buoyant turbulent mixing in shear layers

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1 Introduction

Buoyancy effects in unstably stratified mixing layers express themselves through gravity currents of heavy fluid which propagate in an ambient lighter fluid. These currents are encountered in numerous geophysical flows, industrial safety and environmental protection issues [1]. During transition to turbulence a strong distortion of the separating interface between regions containing ‘heavy’ or ‘light’ fluid arises. The complexity of this interface will be used to monitor the progress of the mixing. We concentrate on the enhancement of surface-area and ‘surface-wrinkling’ of the separating interface as a result of gravity-effects.

Mixing in a turbulent flow may be visualized by a number of snapshots, e.g., in an animation. However, for a quantitative analysis, more specific measures are needed. Here we represent the separating interfaces as level-sets of a scalar field c ($0 \leq c \leq 1$). We establish a close correspondence between the growth of surface-area and wrinkling, in particular at large buoyancy effects. We also show that this process can be simulated quite accurately using large-eddy simulation with dynamic subgrid modeling. However, the subgrid resolution, defined as the ratio between filter-width Δ and grid-spacing h , should be sufficiently high to avoid contamination due to spatial discretization error effects.

2 Surface-area and wrinkling of level-sets

To specify the intricate structure of the separating interface we consider global properties of the level-set defined by $S(t) = \{\mathbf{x} \mid c(\mathbf{x}, t) = 1/2\}$. The mixing-efficiency η is defined in terms of the surface-area A as:

$$\eta(t) = \frac{A(t)}{A(0)} \quad ; \quad A(t) = \int_{S(t)} dS = \int_V d\mathbf{x} \, \delta(c(\mathbf{x}, t) - \frac{1}{2}) |\nabla c(\mathbf{x}, t)| \quad (1)$$

Instead of integrating directly over the complex and possibly fragmented interface $S(t)$, we evaluate the volume integral in 1, in which $S \subset V$, using a locally

exact quadrature method developed in [2]. A further characterization of the geometry is obtained by considering the average curvature C and wrinkling W :

$$C(t) = \int_{S(t)} dS \kappa(\mathbf{x}, t) \quad ; \quad W(t) = \int_{S(t)} dS |\kappa(\mathbf{x}, t)| \quad (2)$$

where the local curvature κ is defined as

$$\kappa(\mathbf{x}, t) = \nabla \cdot \mathbf{n} \quad ; \quad \mathbf{n}(\mathbf{x}) = \frac{\nabla c(\mathbf{x})}{|\nabla c(\mathbf{x})|} \quad (3)$$

The effect of gravity is represented by source terms in the momentum and energy equations [2]. Within the Boussinesq approximation we have:

$$\begin{aligned} \partial_t \rho + \partial_j (\rho u_j) &= 0 \\ \partial_t (\rho u_i) + \partial_j (\rho u_i u_j) + \partial_i p - \partial_j \sigma_{ij} &= - \left(\frac{\alpha \rho c}{Fr^2} \right) \delta_{i2} \\ \partial_t e + \partial_j ((e + p) u_j) - \partial_j (\sigma_{ij} u_i) + \partial_j q_j &= - \left(\frac{\alpha \rho c}{Fr^2} \right) u_2 \end{aligned} \quad (4)$$

where ρ denotes the ‘light’ fluid density, u_i is the velocity in the x_i direction, p the pressure, e the total energy density, q_j the heat-flux and $\sigma_{ij} = S_{ij}/Re$ with S_{ij} the rate of strain tensor and Re the Reynolds number. The source terms on the right hand side represent gravity effects in the negative vertical (i.e. x_2) direction in which $\alpha = (\rho_h - \rho)/\rho$ with ρ_h the density of the ‘heavy’ fluid. The Froude number $Fr = U^*/\sqrt{g^* L^*}$ with g^* the gravitational constant and U^* , L^* a reference velocity and length scale respectively. We consider $\alpha = 1$. Finally, c denotes a scalar field which relates to the local density $\tilde{\rho}$ as $\tilde{\rho} = (1 - c)\rho + c\rho_h$. The scalar field c is assumed to evolve according to

$$\partial_t (\rho c) + \partial_j (u_j \rho c) - \frac{1}{Sc} \partial_{jj} c = 0 \quad (5)$$

where Sc denotes the Schmidt number. The scalar field c is convected by the flow field u_j but in turn affects the flow through the source terms in 4. The initial state for c is a step from a value 0 in the lower half to 1 in the upper half of the domain, representing a heavy fluid layer on top of a lighter one.

3 Buoyancy effects in turbulent mixing

To quantify buoyancy effects in turbulent mixing we consider the temporal mixing layer as documented in [4]. The flow starts from a perturbed laminar profile. A fourth order accurate finite volume discretization was used. Rapid transition to turbulence emerges, displaying helical pairings. At large buoyancy effects the separating interface becomes very distorted and protruding heavy/light fluid ‘fingers’ with a complex geometrical structure emerge, as shown in figure 1(a).

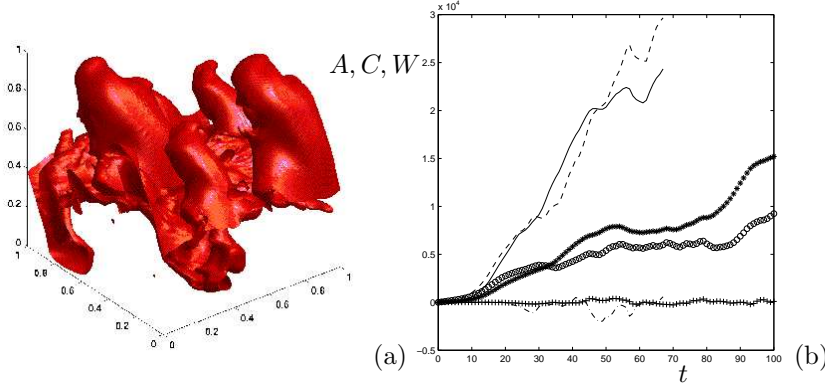


Figure 1: Separating interface at $t = 40$ using $Fr = 2$ (a); surface-area, wrinkling and curvature (b) at $Fr = 2$: $A(t) - A(0)$ (solid), W (dashed), C (dash-dotted) and $Fr = 5$: $A(t) - A(0)$ (*), W (o), C (+). Resolution of 128^3 cells; $Sc = 10$.

The geometrical properties A , C and W of the separating interface are compared in figure 1(b) for two different Froude numbers. The surface-area and wrinkling display approximately the same behavior, while the average surface-curvature remains quite limited, indicating that regions with negative and positive local curvature are about equally important in this flow.

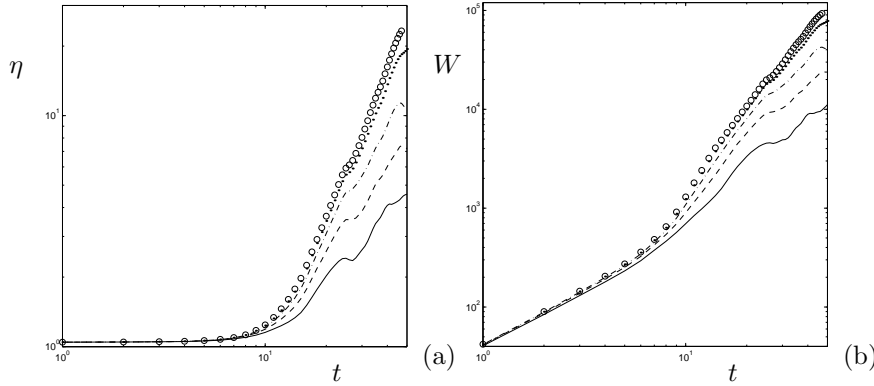


Figure 2: Mixing-efficiency η (a) and surface wrinkling (b) at $Fr = 2$ and $Sc = 5$ (solid), 10 (dashed), 20 (dash-dotted), 50 (dotted), 100 (o). Resolution: 128^3 .

In figure 2 we compare η and W for various Sc . An increase in Sc reduces dissipation in 5 and results in enhanced mixing. After a laminar development up to $t \approx 10$, a nearly algebraic growth $\eta \sim t^\beta$ emerges with $\beta = \beta(Sc)$. The wrinkling also develops algebraically. Initially this development is independent of Sc . Beyond $t \approx 10$ the algebraic growth-rate increases with Sc .

We also considered LES of this mixing flow. In figure 3 we compare pre-

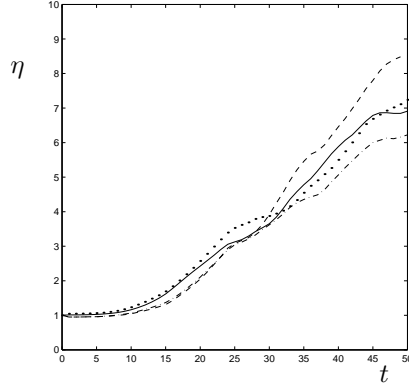


Figure 3: Mixing-efficiency η at $Fr = 2$ and $Sc = 10$: DNS (solid), no-model 32^3 (dashed), dynamic mixed model (32^3 : dash-dotted, 64^3 : dotted); $\Delta = L/16$.

dictions from DNS and LES with the dynamic mixed model. The filter-width $\Delta = L/16$ with L the side of the flow domain. The mixing-efficiency is slightly under-predicted on 32^3 cells. The discrepancy with DNS is largely of numerical nature. By increasing the subgrid-resolution $r = \Delta/h$ from 2 on 32^3 to 4 on 64^3 a strong improvement arises even though Δ was kept fixed. The remaining errors are more representative of subgrid modeling deficiencies [3].

4 Concluding remarks

Buoyancy can strongly increase turbulent mixing. A reasonable agreement between surface-area and wrinkling was observed. This may be relevant, e.g., in combustion modeling to capture flame-speed and intensity. Prediction of mixing-efficiency using LES with dynamic models, showed a large influence of spatial discretization errors. A subgrid-resolution $r = 4$ is suggested.

References

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